Journal of Sound and Vibration (1997) **201**(1), 137–144

TIME AND FREQUENCY DOMAIN PROCEDURE FOR IDENTIFICATION OF STRUCTURAL DYNAMIC MODELS

L. D. Peterson
University of Colorado, Boulder CO 80309, *U.S.A.*

and

K. F. Alvin
Sandia National Laboratories, Albuquerque, NM 87185, *U.S.A*

(*Received* 20 *May* 1994, *and in final form* 21 *June* 1996)

1. INTRODUCTION

Obtaining accurate estimates of modal parameters from test data is an essential part of designing and analyzing aerospace structures. As both the precision required of structural models and the modal complexity of the structures themselves have increased, it has become more challenging to obtain accurate estimates of modal parameters. A central part of this challenge is improving the numerical procedure with which the modal parameters are extracted from the data. Most current research has focused primarily on time domain procedures such as Eigensystem Realization Algorithm (ERA) [1] and the polyreference algorithm [2] to accomplish the modal parameter extraction. Although each performs well on simulated data sets, none has been shown to have a distinct advantage when applied to actual modal test data. The major shortcoming is that high time domain accuracy of the curve fit is not well reflected in frequency domain accuracy, and may lead to large errors in the frequency domain near anti-resonances.

In this note, we present a frequency domain curve fit algorithm for estimating the modal vectors associated with the modes estimated using time domain realization methods. We have found in previous work that significant increases in the amount of data used in time domain realization algorithms does not significantly improve frequency domain error near the anti-resonances. It does, however, converge the poles of the structure very tightly. This feature is exploited in the present procedure.

Several researchers have also investigated similar methods. Reference [3] presents a procedure which, beginning with an identified ERA model, performs a full non-linear minimization in which poles, residuals and modal vectors are simultaneously adjusted. In contrast, reference [4] presents a procedure which, using poles estimated from the polyreference method, performs a modal vector adjustment on each mode individually using data near the resonances. Finally, reference [5] uses a matrix fraction decomposition approach to find both the poles and the modal vectors, but reduces the resulting model with a final ERA identification. Our approach is to use the converged pole estimates from an asymptotically large ERA analysis and then find the output modal vectors and residuals in a separate, overdetermined linear least squares problem. In this way, it is similar philosophically to the method of reference [4], but finds a simultaneous solution for the modal vectors and residuals. Also, it avoids the computational burdens of the non-linear minimization problem used in reference [3] by finding the poles in a separate ERA procedure.

In the following sections, we present and illustrate the method on experimental data. First, the method by which poles are identified in the time domain is discussed, relying

0022–460X/97/110137 + 08 $25.00/0/sv960692

primarily on references to prior work. Then, the necessary discrimination of spurious poles from the identified set is discussed. The linear least squares problem for modal vectors and residuals is discussed, following by an application of the method to experimental data.

2. TIME DOMAIN POLE IDENTIFICATION

The response $y(t)$ of a structure to a set of forces or inputs $u(t)$ can be modelled as a finite order discrete state space model of the form

$$x(k+1) = Ax(k) + Bu(k), \qquad y(k) = Cx(k) + Du(k), \tag{1}$$

in which k is the time sample index. All state space time domain realization methods attempt to find the state space matrices A, B, C and D from measurements of the Markov parameter sequence. This is the process known as *system realization*. Given that frequency response functions $G(j\omega)$ have been estimated from the measured data, the Markov parameters $M(k)$ are given by the inverse discrete Fourier transform of $G(j\omega)$. The essential considerations in system realization are the selection of the model order (it is presumed that the model form is correct) and the determination of the state space parameters from a minimization of some prediction error. For ERA, the prediction error is defined in terms of a Hankel matrix of the Markov parameters. The ERA realization finds the linear least squares solution for A, B, C and D to minimize the error in the shift in the Hankel matrix of the system model and the data. The model order is selected by examining the numerical rank of the Hankel matrix formed from the data. Reference [6] discussed the efficient computation of the ERA realization for data sets sufficiently large to converge the pole estimates; reference [7] examined the variance of ERA pole estimates as a function of data set size, demonstrating the data set size necessary to achieve convergence.

3. DISCRIMINATION OF POLES FROM TIME DOMIAN IDENTIFICATION

Our procedure uses a combination of several quantitative modal quality indicators (MQI) to detect convergence and discriminate unwanted or unreliable modes from the A and B matrices which are subsequently used in the frequency domain curve fit. We note that none of these indicators is demonstrably superior over the others. In our experience, only a careful, combined examination of all of these MQI leads to reliable discrimination of structural poles: (1) Modal Singular Value (MSV) [8], (2) Extended Modal Amplitude Coherence (EMAC) [9] and (3) Consistent Mode Indicator (CMI) [9].

In addition to the above, we have also developed a new indicator. The Mass Scaling Parameter (MSP) evaluates the extent to which the identified mode shape can be normalized by the driving point measurement collocated with the applied forces. If C_{ij} is the displacement mode shape value for mode i at DOF j, $C_{i\max}$ is the highest magnitude modal displacement for mode i, and B_{ij} is the continuous time modal participation factor of force for mode i at DOF j, then

$$MSP = |C_{imax}/C_{ij}| \operatorname{sgn} \frac{B_{ij}}{C_{ij}}. \tag{2}$$

Modes with small components at the driving points are poorly excited and poorly normalized. In addition, a negative sign change between the B_{ij} and the C_{ij} indicates that the mode has negative modal mass. By discriminating modes with low or negative values of MSP, one can exclude modes for which the extracted normalized mode shape is unreliable.

4. FREQUENCY DOMAIN IDENTIFICATION PROCEDURE FOR MODAL VECTORS AND RESIDUALS

A fundamental limitation of discrete time domain state space realization algorithms for continuous systems is that they cannot accurately represent the influence of modes above the Nyquist frequency in an efficient form. That is, the effects of these so-called residual modes on the response functions leads to residual terms in the continuous model which do not have a corresponding finite representation in the discrete model. In classical structural dynamics theory, the predominate residual terms are residual flexibility and residual mass. It is well known that neglecting these terms can lead to poor frequency domain accuracy for an identified model, even when the modes within the test bandwidth are perfectly identified. Herein lie many of the aforementioned problems with time domain realization algorithms; in particular, ambiguity in model order and poor frequency domain reconstruction for the resultant model. Unlike noise, an error in the model form cannot be mitigated by averaging more data or changing the error norm which serves as the basis of the numerical algorithm.

The present procedure recognizes this limitation of discrete state space realization algorithms and proposes to add a second stage linear estimation to determine the modal vectors and residual terms. The procedure exploits the primary strength of the existing realization algorithms; that is, their ability to identify extremely accurate estimates of all the resonant poles in the data via efficient and robust linear numerical analysis. Using this capability, together with the MQI previously discussed to discriminate the estimated poles, the second stage procedure assumes that the matrices A and B of the identified model are known.

Assuming acceleration output and force input, we model the continuous response of the system as the sum of the response of the identified modes $G^n_{ik}(s)$ and the contribution of the residuals $R_{ik}(s)$; viz.,

$$\hat{G}_{ik}(s) = G^n_{ik}(s) + R_{ik}(s), \tag{3}$$

where

$$G^n_{ik}(s) = \sum_{n=1}^{N_{ID}} \frac{s^2 \Phi_{in}\Phi_{kn}}{s^2 + 2\zeta_n\omega_n s + \omega_n^2}, \qquad R_{ik}(s) = \sum_{n=N_{ID}+1}^{\infty} \frac{s^2 \Phi_{in}\Phi_{kn}}{s^2 + 2\zeta_n\omega_n s + \omega_n^2}. \tag{4}$$

Then expanding the residual term $R_{ik}(s)$ with the assumption that $s \ll j\omega_n$, we have

$$\begin{aligned} R_{ik}(s) &= s^2 \sum_{n=N_{ID}+1}^{\infty} \frac{\Phi_{in}\Phi_{kn}}{\omega_n^2} - s^4 \sum_{n=N_{ID}+1}^{\infty} \frac{\Phi_{in}\Phi_{kn}}{\omega_n^4} + \ \ldots \\ &= s^2 F_{ik} - s^4 H_{ik} + \ \ldots, \end{aligned} \tag{5}$$

where

$$F_{ik} = \sum_{n=N_{ID}+1}^{\infty} \frac{\Phi_{in}\Phi_{kn}}{\omega_n^2}, \qquad H_{ik} = \sum_{n=N_{ID}+1}^{\infty} \frac{\Phi_{in}\Phi_{kn}}{\omega_n^4}. \tag{6}$$

In terms of the discrete-time state space model $[A, B, C, D]$ and the residuals, our representation of the FRF at a particular frequency f_k becomes (for different types of output):

acceleration: $\hat{G}(f_k) = C(e^{j2\pi f_k \Delta t} I - A)^{-1} B + D - (2\pi f_k)^2 F - (2\pi f_k)^4 H;$

velocity: $\hat{G}(f_k) = C(e^{j2\pi f_k \Delta t}I - A)^{-1}B + D + j(2\pi f_k)F + j(2\pi f_k)^3 H;$

displacement: $\hat{G}(f_k) = C(e^{j2\pi f_k \Delta t}I - A)^{-1}B + F + (2\pi f_k)^2 H.$ (7)

Therefore, to estimate the output quantities C, D, F and H for acceleration measurements, we can write the linear equation

$$\hat{G}(f_k) = [C \quad D \quad F \quad H]\begin{bmatrix} (e^{j2\pi f_k \Delta t}I - A)^{-1}B \\ I \\ -(2\pi f_k)^2 I \\ -(2\pi f_k)^4 I \end{bmatrix} = [C \quad D \quad F \quad H]S(f_k). \tag{8}$$

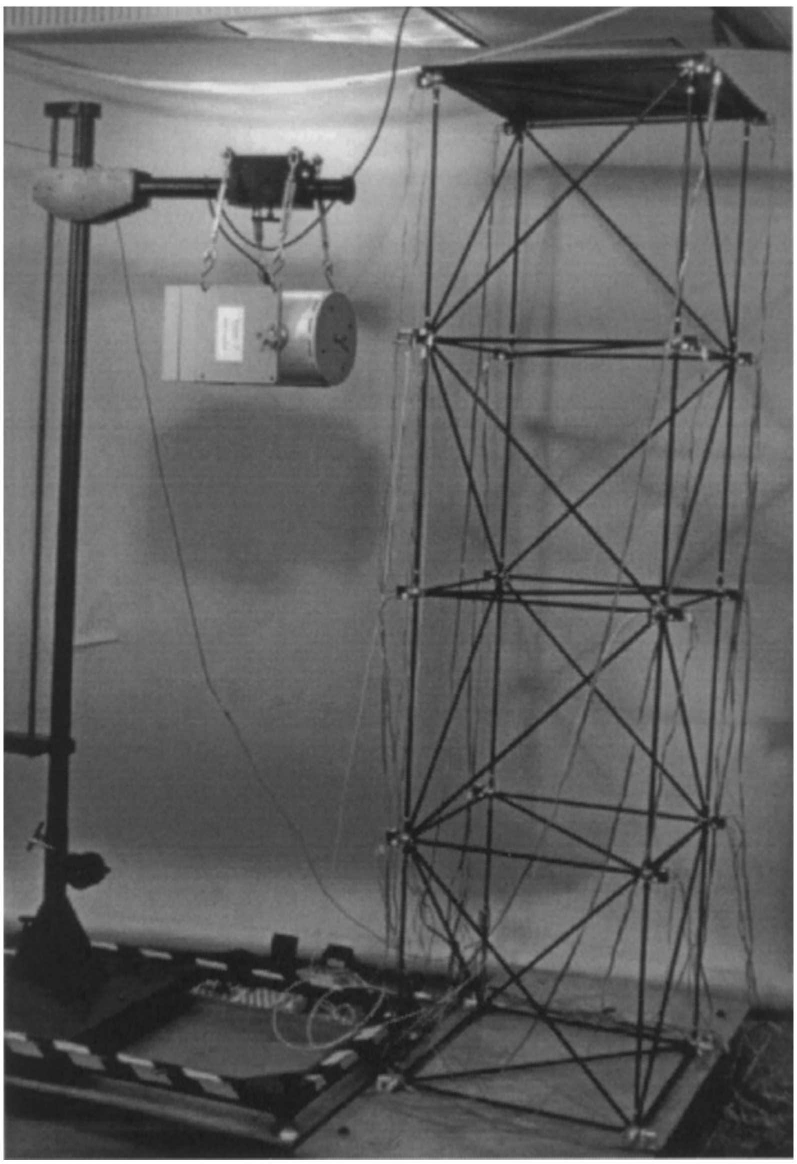

Figure 1. The four-bay cantilevered truss used in this research.

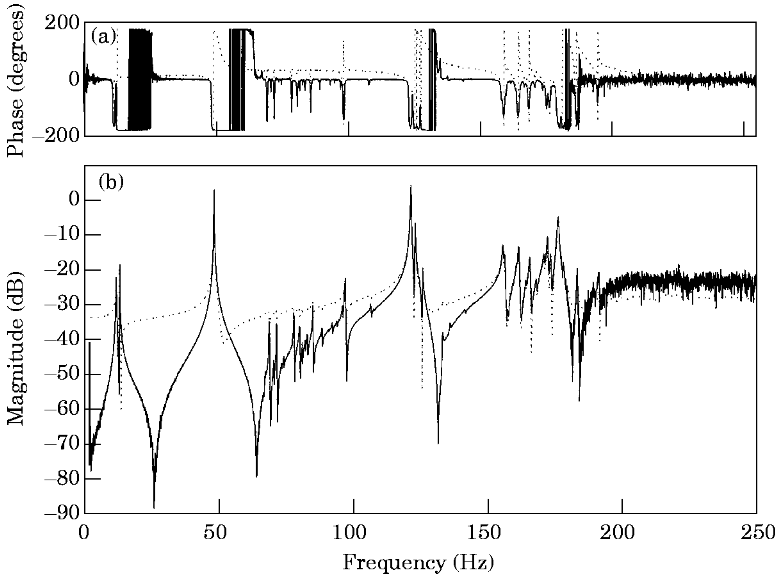

Figure 2. A reconstruction of the driving point FRF for the initial 200-mode (400-state) ERA model: (a) phase, (b) magnitude. ——, Measured; · · · , identified ERA model.

In this form, the error minimization problem is given by

$$\min_{[C\ D\ F]} \sum_{k \in k_{set}} \| G(f_k) - \hat{G}(f_k) \|^2 \tag{9}$$

and the solution is given by

$$[C \quad D \quad F \quad H] = \left\{ \sum_{nk \in k_{set}} G(f_k) S(f_k)^* \right\} \left\{ \sum_{k \in k_{set}} S(f_k) S(f_k)^* \right\}^{-1}, \tag{10}$$

in which $k \in k_{set}$ denotes the set of frequency samples to be included in the curve fit. A pseudo inverse is generally unnecessary, as rich data typically results in a full rank matrix that can be inverted directly. Frequency weights could also be included proportionately to emphasize frequency bands of interest, but we have found that this is usually not necessary.

5. EXAMPLE APPLICATION

The testbed used to illustrate our modal identification procedure is shown in Figure 1. It is a cantilevered truss consisting of four 0·5 bays with a 50 lb steel plate tip mass. FRF data was collected using a pseudo-random force input applied with an electromagnetic shaker in voltage-coupled mode. Auto- and cross-spectra were averaged for 50 ensembles consisting of 8192 samples collected at a rate of 500 Hz. A total of 62 accelerometer measurements were collected, including a driving point measurement for mass normalization of the mode shapes. The input signal was band limited to 240 Hz and the sensors were appropriately filtered to avoid aliasing of signals above the Nyquist frequency.

For the time domain identification phase, an ERA analysis was performed with a Hankel matrix of 40 block rows and 4000 block columns. Models with orders ranging from 50 to 400 states were extracted from the ERA analysis, and the convergence of the model poles with respect to the MQI were studied. In Figure 2 is shown the reconstruction of the driving point data for the 400 state ERA model. From this analysis, 124 modes were identified. In Table 1 is shown the MQI for the first 30 modes identified by the ERA analysis. The far right column indicates the modes which are chosen for subsequent analysis.

The second stage frequency domain identification procedure was then applied. Although this is a direct linear least-squares problem, it is wise to re-apply the mode discrimination procedure after re-computing the modal vectors, in order to detect additional unreliable modes. Using this, the initial 124 mode discriminated ERA model was reduced first to 58 modes and finally to 42 modes at the end of this procedure. The final curve fit, which includes the residual flexibility term, is shown in Figure 3. The modal vectors and residuals were computed using the above frequency domain curve including data up to 120 Hz. Compare the accuracy of this 42-mode reconstruction to that of the 200-mode reconstruction shown in Figure 2 using the original ERA model. Note that the curve fit accuracy is very high within the bandwidth of the curve fit, and maintains reasonable accuracy for modes outside the bandwidth. Note also that the phase error is also greatly

Table 1

Modal quality indicators for the ERA identified modes (124-*mode model*)

ERA mode	f (Hz)	ζ_i (%)	EMAC (%)	MSV (%)	MPC (%)	MSP (%)	Identified mode
1	0·0	86·5	94·9	16·5	96·0	15·3	
2	0·9	6·8	78·3	3·6	38·2	2·8	
3	11·5	0·8	98·8	23·5	100	64·0	1
4	12·8	0·2	99·1	20·7	99·8	48·9	2
5	12·9	6·8	50·7	4·0	74·6	−26·1	
6	47·9	0·1	94·9	71·3	99·1	68·4	3
7	48·0	0·1	97·2	47·0	24·6	−2·9	
8	52·9	0·5	73·0	6·2	97·4	−0·6	
9	66·6	0·1	87·8	7·9	89·7	−3·2	
10	66·8	0·1	91·6	8·7	98·0	−2·2	
11	68·3	0·1	96·9	14·7	98·0	19·9	4
12	68·9	0·4	72·2	5·0	88·1	8·6	5
13	70·0	0·1	91·7	9·7	99·2	4·3	6
14	71·0	0·1	94·8	18·7	43·6	12·1	
15	71·1	0·2	90·4	13·2	59·3	7·2	7
16	73·0	0·2	83·0	6·8	97·1	2·2	8
17	73·8	0·1	87·8	3·6	98·3	2·8	9
18	74·5	0·3	68·7	23·5	82·3	64·0	
19	77·0	0·2	75·7	20·7	95·4	48·9	
20	77·6	0·1	96·5	4·0	99·4	−26·1	10
21	79·5	0·4	90·9	71·3	78·3	68·4	11
22	79·7	0·1	95·2	47·0	79·3	−2·9	12
23	80·6	0·1	93·6	6·2	97·1	−0·6	13
24	81·7	0·1	90·5	7·9	97·4	−3·2	14
25	82·6	0·1	93·8	8·7	98·8	−2·2	15
26	84·6	0·1	96·0	14·7	95·2	19·9	16
27	85·1	0·1	83·9	5·0	90·8	8·6	17
28	85·3	0·6	63·6	9·7	66·3	4·3	
29	87·7	0·1	89·9	18·7	99·6	12·1	18
30	88·1	0·1	96·6	13·2	98·0	7·2	19

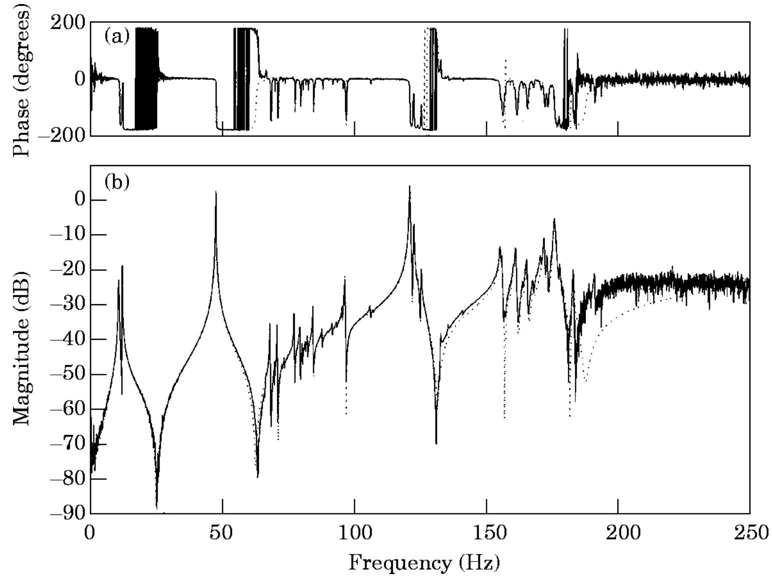

Figure 3. Data reconstruction after a frequency domain curve fit with residuals and reduction to 42 modes (84 states).

reduced over the entire band, and the troublesome phase lead error at intermediate frequencies is nearly zero.

6. CONCLUSIONS

The method reported in this note obtains a minimal order modal model with both time and frequency domain accuracy using only overdetermined linear regressions. The converged poles from an overdetermined ERA model are carefully discriminated using a combination of modal quality indicators to eliminate redundant or non-physical modes. The model vectors are then determined using a linear least-squares curve fit to the FRF data, which includes residual terms. These additional terms compensate for the fact that a discrete state space model cannot represent the effects of super-Nyquist modes on the FRF, although most structural modal test data is significantly influenced by their low frequency residual dynamics.

ACKNOWLEDGMENTS

This research was sponsored by Sandia National Laboratories under contract AJ-4223, NASA/LaRC under contract NAG-1-1490, and by a donation from the Shimizu Corporation.

REFERENCES

1. J. N. JUANG and R. S. PAPPA 1985 *Journal of Guidance, Control and Dynamics* **8**, 620–627. An eigensystem realization algorithm for modal parameter identification and model reduction.
2. H. VOLD, J. KUNDRAT, G. T. ROCKLIN and R. RUSSELL 1982 *SAE Paper* 820194. A multi-input modal estimation algorithm for mini-computers.
3. R. N. JACQUES and D. W. MILLER 19xx *32nd IEEE Conference on Decision and Control*. Multivariable model identification from frequency response data.
4. R. L. MAYES 1993 in *Proceedings of the 11th International Modal Analysis Conference, February*. A multi-degree-of-freedom mode shape estimation algorithm using quadrature response.

5. L. Horta and J.-N. Juang 1993 in *Proceedings of the AIAA Guidance*, *Navigation and Controls Conference*, *Monterey*, *CA*, 9–11 *August*. 1236–1242. Frequency domain system identification methods: matrix fraction description approach.
6. L. D. Peterson 1995 *Journal of Guidance*, *Control and Dynamics* **18**, 395–403. Efficient computation of the eigensystem realization algorithm.
7. L. D. Peterson, S. J. Bullock and S. W. Doebling *International Journal of Analytical and Experimental Modal Analysis*. The statistical sensitivity of experimental modal frequencies and damping ratios to measurement noise.
8. R. W. Longman, J.-S. Lew and J.-N. Juang 1992 *AIAA Paper* 92-2518-CP; in *Proceedings of the* 1992 *AIAA Structures*, *Structural Dynamics*, *and Materials Conference*, *April*, 2307–2317. Comparison of candidate methods to distinguish noise modes from system modes in structural identification.
9. R. S. Pappa, K. B. Elliot and A. Schenk 1992 *AIAA Paper* 92-2136-CP; in *Proceedings of the* 1992 *AIAA Structures*, *Structural Dynamics*, *and Materials Conference*, *April*. 556–565. A consistent-mode indicator for the eigensystem realization algorithm.
10. D. J. Ewins 1984 *Modal Testing*: *Theory and Practice*. Letchworth, U.K: Research Studies Press. See pp. 170–177.